DEI - Università di Padova
ACG
Advanced Computing Group

Graph Models of Information Spreading in Wireless Networks

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XXIV Series — ICT Curriculum

Committee: Prof. R. Giancarlo, Prof. A. Panconesi, Prof. D. Rocchesso (chair)

Wireless Networks

Mobile Devices Networks

Vehicular Networks

Wildlife Surveillance Systems

Field Operations

Motivation

Ad hoc/sensor/vehicular/personal networks will be the future of distributed computing (as efficiency + integration $\uparrow$, cost-per-unit $\downarrow$)

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So far simulation- or modeling-driven development process $\Rightarrow$ expensive and heavily technology-dependent

We need theoretical knowledge of the fundamental properties of these systems, in order to design efficient and scalable algorithms

Thesis Objectives and Results

Objectives

- Define graph models of interesting networks
- Analyze their properties related to information spreading
- Design efficient algorithms using this knowledge

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Results

Characterization of:

- Expansion and diameter of Bluetooth networks
- Flooding time in dynamic Bluetooth networks
- Broadcast time in sparse mobile networks and several related scenarios

Outline of the Presentation

- Bluetooth Networks
  - Bluetooth Topology Model
  - Expansion and Diameter
  - Flooding Time in Dynamic BT Networks
- Dynamic Graphs
  - Random Walker Model
  - Broadcast Time
  - Related Scenarios
- Conclusions

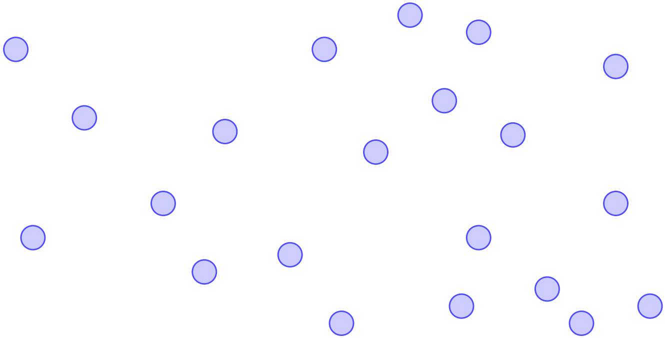
Bluetooth Networks

Bluetooth Technology

- Technology for wireless communication introduced as cable replacement for small PANs connecting laptops, mobile phones, PDAs, etc.
- Arguments in favor of BT for large ad-hoc scenarios:
  - cheap and easily integrable
  - good data rate/energy consumption tradeoff
  - wide adoption

Bluetooth Technology: Network Organization/Formation

- Piconet: 1 master, $\leqslant 7$ slaves
- Scatternet: interconnection of piconets through gateways to form multi-hop ad hoc network; three phases: device discovery, piconet formation, scatternet formation



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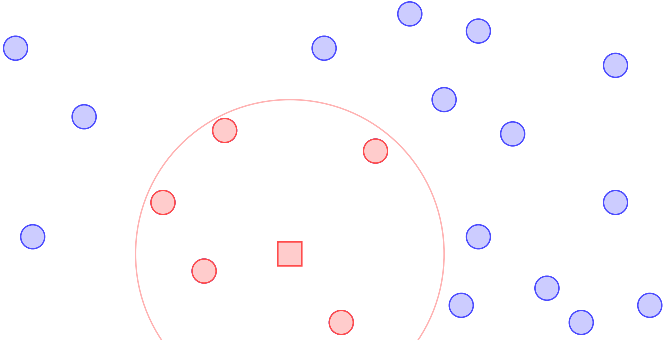

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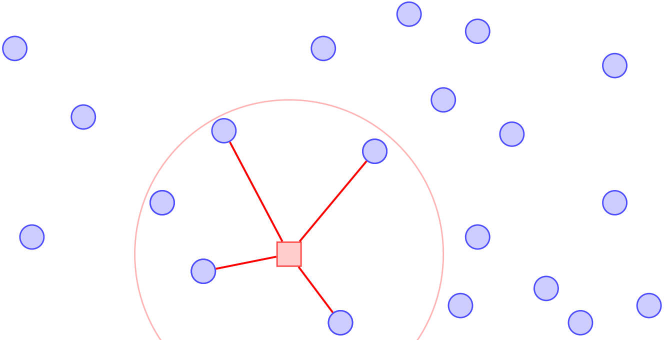

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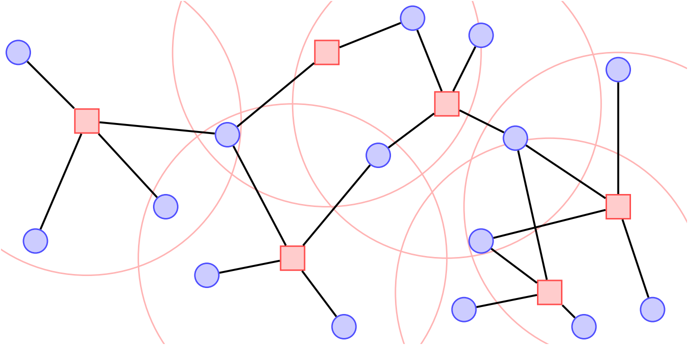

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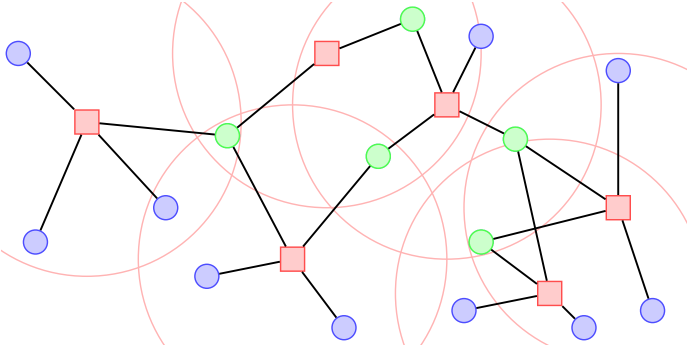
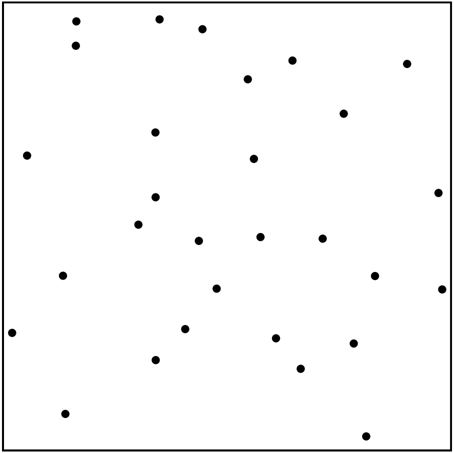


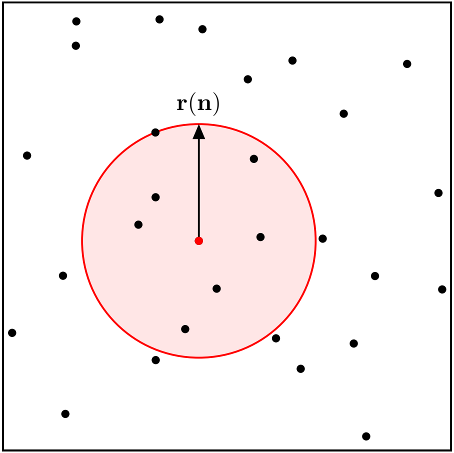
Bluetooth Topology: Mathematical Model



Graph $BT(r(n), c(n))$

- n nodes (*devices*) placed at random in $[0, 1]^2$

Bluetooth Topology: Mathematical Model



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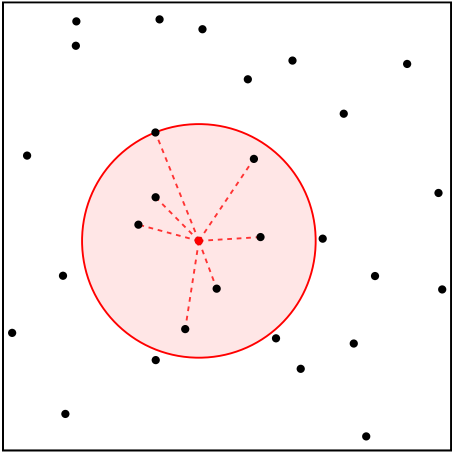

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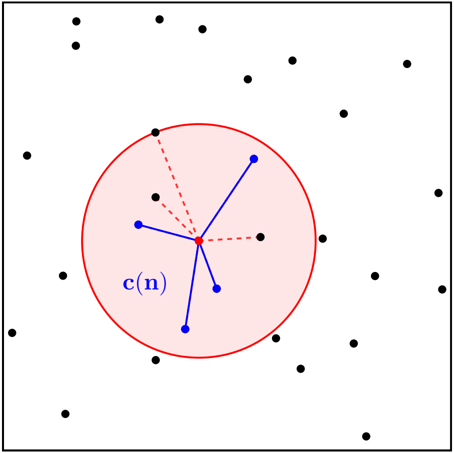
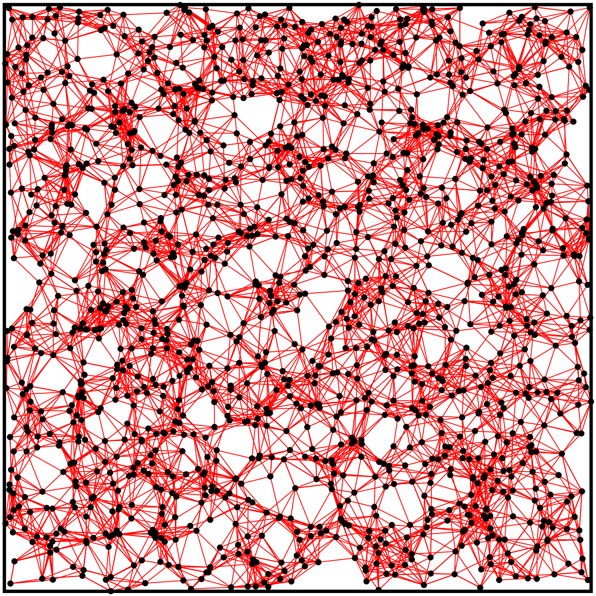


Graph $BT(r(n), c(n))$

- n nodes (*devices*) placed at random in $[0, 1]^2$
- visibility range $r(n)$
- among all visible nodes each device independently selects $c(n)$ random neighbors (it selects all visible nodes if $< c(n)$)



$BT(0.075, 5)$ with $n = 1500$ nodes.

Relevant Questions

How many neighbors should each device discover, in order for BT to exhibit:

- connectivity (i.e., single connected component)?
- good expansion (i.e., high bandwidth)?
- low diameter (i.e., low latency)?

Previous Work

- [Penrose 03]: $r(n) = \Omega\left(\sqrt{\log n/n}\right)$ necessary and sufficient to achieve connectivity w.h.p., when each node connects to *all* visible nodes (Random Geometric Graph or visibility graph)
- [Panconesi et al., 04]: for $r(n) = \Theta(1)$, $c(n) = \Theta(1)$ suffices to attain high expansion w.h.p.
- [Dubhashi et al., 05]: for $r(n) = \Theta(1)$, $c(n) = 2$ suffices to attain connectivity w.h.p.

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Remark

Setting $r(n) = \Theta(1)$ implies that each node *sees* a constant fraction of all other nodes $\Rightarrow$ unfeasible for large n.

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Remark

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Analysis for $r(n)$ decreasing in n is needed!

Previous Work (cont'd)

Theorem (Crescenzi, Nocentini, Pietracaprina, Pucci, 2007)

There exist two positive real constants γ_1, γ_2 *such that if*

$$r(n) \geqslant \gamma_1 \sqrt{\frac{\log n}{n}} \quad \textit{and} \quad c(n) = \gamma_2 \log \frac{1}{r(n)}$$

then $BT(r(n), c(n))$ *is connected w.h.p.*

Our Contribution [ESA'09, ToCS'12]

- Tight bounds on the expansion of $\mathrm{BT}(r(n), c(n))$
- Tight bounds (up to a logarithmic additive term) on the diameter of $\mathrm{BT}(r(n), c(n))$
- An upper bound on the flooding time in a dynamic version of $\mathrm{BT}(r(n), c(n))$, where nodes move over time

Expansion of BT$(r(n), c(n))$

Definition (Node Expansion)

$$\lambda(s) = \min_{S \subseteq V : |S| = s} \frac{|\Gamma(S) - S|}{s}, \quad 1 \leqslant s \leqslant |V|/2.$$

Theorem (Expansion of BT)

Let $m = \Theta\left(nr^2(n)\right)$. Then, there exist two constants $\gamma_1, \gamma_2 > 0$ s.t. if

$$r(n) \geqslant \gamma_1 \sqrt{\frac{\log n}{n}} \quad \textit{and} \quad c(n) = \gamma_2 \log \frac{1}{r(n)}$$

then the expansion of BT$(r(n), c(n))$ is, w.h.p.,

$$\lambda(s) = \begin{cases} \Theta\left(\min\{c(n), m/s\}\right) & \textit{if} \quad 1 \leqslant s \leqslant \alpha m \\ \Theta\left(\sqrt{m/s}\right) & \textit{if} \quad \alpha m < s \leqslant n/2. \end{cases}$$

Diameter of $\mathrm{BT}(r(n), c(n))$

Definition (Diameter)

$$\mathrm{diam}(G) = \max\{\,\mathrm{dist}(u, v) \;:\; u, v \in V(G)\,\}$$

Theorem (Diameter of BT)

There exist two positive real constants γ_1, γ_2 such that if

$$r(n) \geqslant \gamma_1 \sqrt{\frac{\log n}{n}} \quad \textit{and} \quad c(n) = \gamma_2 \log \frac{1}{r(n)}$$

then the diameter of $BT(r(n), c(n))$ is, w.h.p.,

- $\mathrm{diam}(BT) = O\left(1/r(n) + \log n\right)$
- $\mathrm{diam}(BT) = \Omega\left(1/r(n)\right)$ *(tight for $r(n) = O\left(1/\log n\right)$)*
- $\mathrm{diam}(BT) = \Omega\left(\log n/\log\log n\right)$ *for $r(n) = \Theta\left(1\right)$.*

Dynamic Bluetooth Topology

"Definition" of $\mathcal{G}(n, \rho, r(n), c(n))$

Sequence of Markovian Evolving Graphs $\{G_t\}_{t \in \mathbb{N}}$, where the edge-set of G_t is selected according to the BT$(r(n), c(n))$ protocol, and each node moves in a time step u.a.r. within a ball of radius ρ

Theorem (Flooding Time of DBT)

There exist two positive real constants γ_1, γ_2 such that if

$$r(n) \geqslant \gamma_1 \sqrt{\frac{\log n}{n}} \quad \textit{and} \quad c(n) = \gamma_2 \log \frac{1}{r(n)}$$

then the flooding time of $\mathcal{G}(n, \rho, r(n), c(n))$ is, w.h.p.,

$$T_{FL} = O\left(\frac{1}{r(n)} + \log n\right).$$

Extensions & Open Problems

- Extensions
  - When $r(n) = \Theta\left(\sqrt{\log n/n}\right)$ (minimum radius), $c(n) = \Theta\left(\sqrt{\log n/\log\log n}\right)$ is the minimum number of neighbors needed to achieve connectivity w.h.p. (Broutin et al. [arXiv'11])
  - Generalization to higher dimensions

Extensions & Open Problems

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  - Generalization to higher dimensions
- Open problems
  - Complete characterization of the trade-off between $r(n)$ and $c(n)$
  - Studying how expansion and diameter behave in the above case

Dynamic

Graphs

Mobile Networks: a Closer Look

Mobile networks are distributed systems

- dynamic: topology changes over time…

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- under energy constraints: small transmission radius

Mobile Networks: a Closer Look

Mobile networks are distributed systems

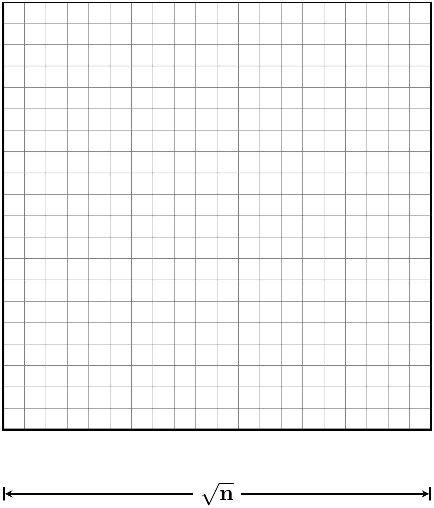
- dynamic: topology changes over time...
- ... but not too fast: mobility speed $\ll$ transmission speed
- with no infrastructure: wireless, multi-hop communications
- under energy constraints: small transmission radius
- essentially planar

Previous Work

- Alves *et al.* [Ann.App.Pr.'02] and Kesten *et al.* [Ann.Pr.'05]
  - shape of the subspace of $\mathbb{Z}^d$ containing "infected" RWs (Frog Model)
- Dimitriou *et al.* [Dis.App.Mat.'06]
  - k agents performing RWs on an n-node graph, bounds on the expected infection time, depending on graph expansion
- Clementi *et al.* [ICALP'09, IPDPS'09]
  - $k = \Theta(n)$ agents on a n-node 2D grid (dense scenario) large maximum speed R and/or large transmission radius r
  - bounds on broadcast time
- Peres *et al.* [SODA'11]
  - Poisson point process in $\mathbb{R}^d$ above percolation threshold agents follow Brownian motion
  - bounds on detection, coverage, broadcast time

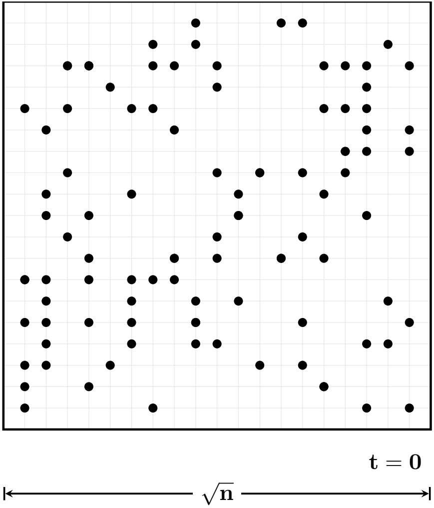
Mobility Model

- $\sqrt{n} \times \sqrt{n}$ 2D grid w/ loops



Mobility Model

- $\sqrt{n} \times \sqrt{n}$ 2D grid w/ loops
- $k = O(n)$ mobile agents
- Initial positions $\equiv$ stationary distribution ($\Rightarrow$ uniform)



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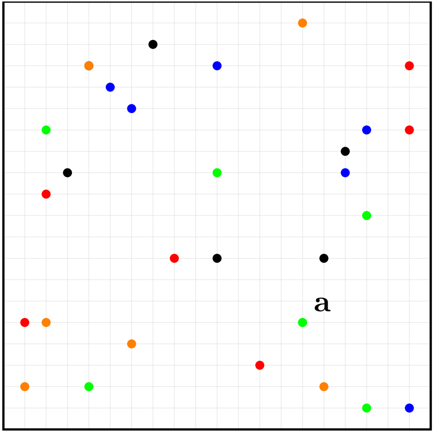
- $pos_a(t) \equiv$ position of agent a at time $t \in \mathbb{N}$



$\mathbf{t = 0}$

Mobility Model

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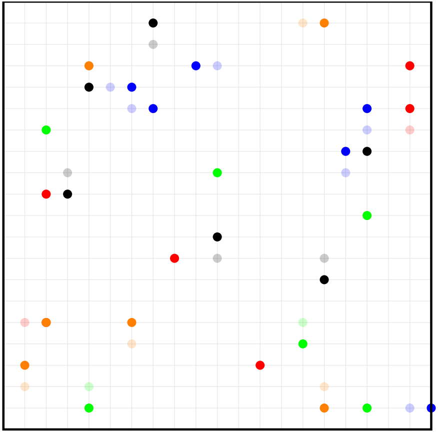

$\mathbf{t = 1}$

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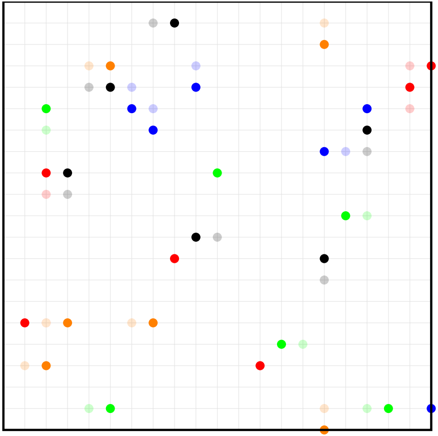

$\mathbf{t = 2}$

Mobility Model

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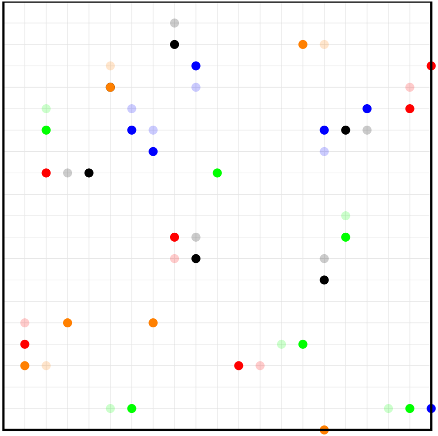

$\mathbf{t = 3}$

Mobility Model

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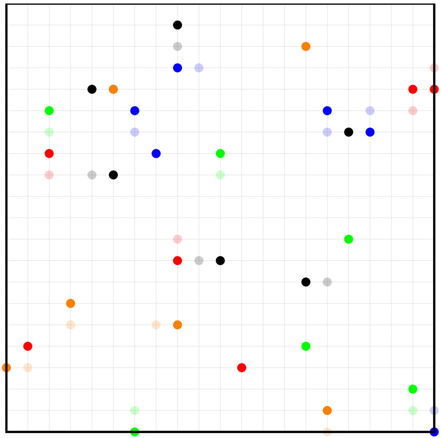

$\mathbf{t = 4}$

Mobility Model

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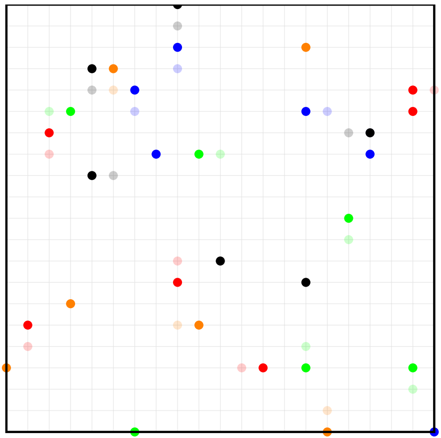
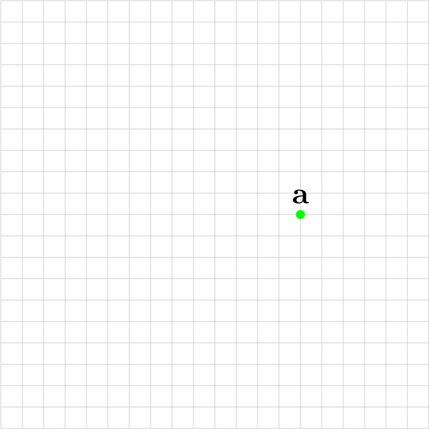


$\mathbf{t = 5}$

Communication Model

- Each agent has transmission radius $r \geqslant 0$



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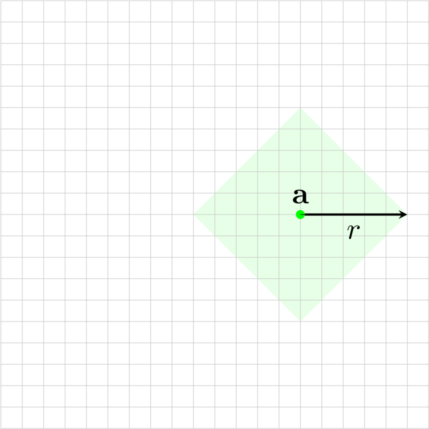

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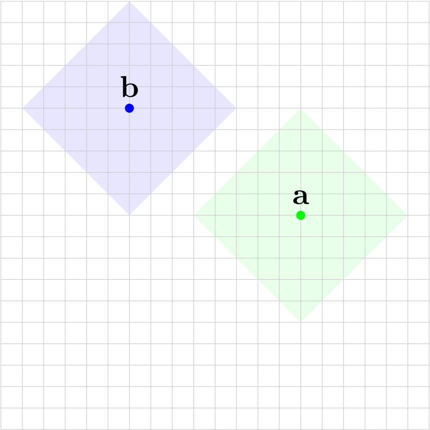

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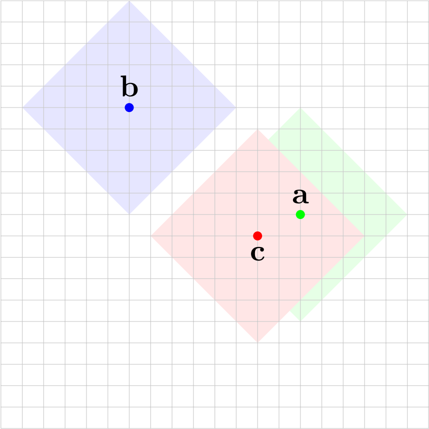
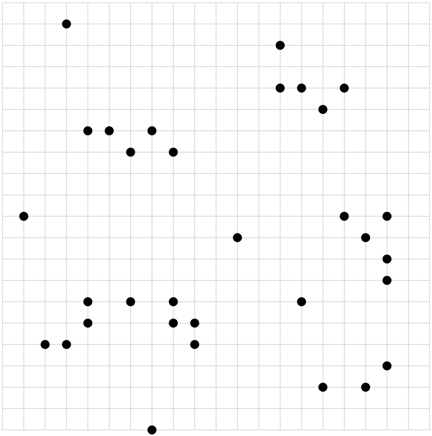


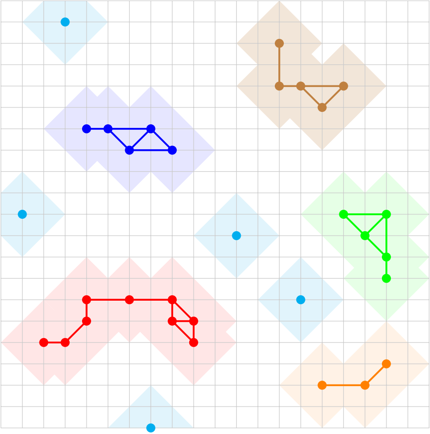
Communication Model

- Each agent has transmission radius $r \geqslant 0$
- Visibility graph $G_t(r)$:
  - vertices $\equiv$ agents



Communication Model

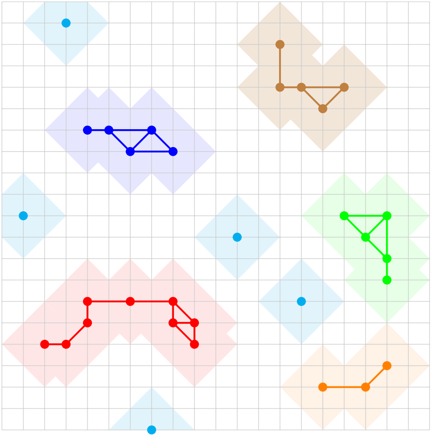
- Each agent has transmission radius $r \geqslant 0$
- Visibility graph $G_t(r)$:
  - vertices $\equiv$ agents
  - edge $\{a, a'\} \in G_t(r) \iff \| pos_a(t) - pos_{a'}(t) \| \leqslant r$
  - each connected component is called "island"



$G_t(r)$

Communication Model

- $M_a(t) \equiv$ messages known by a at time t
- $M_a(t)$ is non-decreasing (agents don't forget messages)
- On a meeting, agents exchange *all* the messages they know



$G_t(r)$

Broadcast Time

Initially, only the source s knows the rumor $\mathcal{M}$:

$$M_s(0) = \{\mathcal{M}\} \qquad \text{and} \qquad M_a(0) = \emptyset \quad \forall a \neq s$$

We study the Broadcast Time T_{B} of the system, which is the first time instant when all the agents know the rumor:

$$T_{\mathrm{B}} = \inf\{\, t \geqslant 0 \,:\, \mathcal{M} \in M_a(t) \quad \forall a \,\}$$

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Remarks

- $T_{\mathrm{B}} \equiv T_{\mathrm{B}}(n, k, r)$
- T_{B} is non-increasing in r: $\qquad r' \geqslant r \Rightarrow T_{\mathrm{B}}(r') \leqslant T_{\mathrm{B}}(r)$
- Broadcast analysis extends to other communication primitives

Our Contribution [PODC'11]

- Tight bounds on the broadcast time T_{B} of a message in a sparse system of mobile agents
- Our analysis techniques extend to several related models (dense case, multiple messages, different interaction rules, ...)

Upper Bound on T_{B}

Theorem 1 (Upper Bound on T_{B})

Let $r = 0$ (physical meetings). Then, for $k \geqslant 2$,

$$T_{\mathrm{B}} = \tilde{O}\left(\frac{n}{\sqrt{k}}\right)$$

with probability $\geqslant 1 - 1/n^2$.

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$$T_{\mathrm{B}} = \tilde{O}\left(\frac{n}{\sqrt{k}}\right)$$

with probability $\geqslant 1 - 1/n^2$.

Since $T_{\mathrm{B}}(r)$ is non-increasing:

Corollary 1

$T_{\mathrm{B}} = \tilde{O}\left(n/\sqrt{k}\right)$ w.h.p. for any $k \geqslant 2$, $r \geqslant 0$.

Quite surprisingly, this bound is essentially tight (see next slide)

Lower Bound on T_{B}

Theorem 2 (Lower Bound on T_{B})

Let $r \leqslant \frac{1}{8e^3}\sqrt{n/k}$. Then, for $k \geqslant 2$,

$$T_{\mathrm{B}} = \tilde{\Omega}\left(\frac{n}{\sqrt{k}}\right)$$

with probability $\geqslant 1 - (2^{-(k-1)} + 1/n + 2/n^2)$.

Lower Bound on T_{B}

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Peres *et al.* [SODA'11] consider the case $r > r_c \approx \sqrt{n/k}$ (i.e., above percolation threshold)

Lower Bound on T_{B}

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$$T_{\mathrm{B}} = \tilde{\Omega}\left(\frac{n}{\sqrt{k}}\right)$$

with probability $\geqslant 1 - (2^{-(k-1)} + 1/n + 2/n^2)$.

Peres *et al.* [SODA'11] consider the case $r > r_c \approx \sqrt{n/k}$ (i.e., above percolation threshold)

Together with Corollary 1, we have the tight result:

Corollary 2

If $k = \Omega(\log n)$ and $r \leqslant \frac{1}{8e^3}\sqrt{n/k}$, then $T_{\mathrm{B}} = \tilde{\Theta}\left(n/\sqrt{k}\right)$ w.h.p.

Extensions & Open Problems

- Our analysis techniques extend to
  - dense scenarios ($k \geqslant n/2$)
  - other communication primitives (gossip, multicast)
  - related models (Frog Model, mobility with jumps, predator-prey)

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  - Modeling barriers and obstacles
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  - Tradeoffs between agent's buffer size and spreading time

Conclusions

Contribution of this Thesis

- Bluetooth Topology
  - Tight bounds on expansion
  - Tight bounds (up to a log additive factor) on diameter
  - Upper bound on flooding time in dynamic Bluetooth networks
- Dynamic Graphs
  - Tight bounds (up to polylog factors) on the broadcast time of a message in sparse mobile networks
  - Our analysis techniques apply to several related scenarios

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List of Publications

- A. P., A. Pietracaprina, G. Pucci
  On the Expansion and Diameter of Bluetooth-Like Topologies, ESA, 2009
- P. Bertasi, A. P., M. Scquizzato, F. Silvestri
  A Novel Resource-Driven Job Allocation Scheme for Desktop Grid Environments, TGC, 2010
- A. P., A. Pietracaprina, G. Pucci, E. Upfal
  Infectious Random Walks, arXiv, 2010
- A. P., A. Pietracaprina, G. Pucci, E. Upfal
  Tight Bounds on Information Dissemination in Sparse Mobile Networks, PODC, 2011
- A. P.
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- A. P., E. Upfal
  Dynamic Line-of-Sight Networks, Work in progress, 2011–2012
- A. P., A. Pietracaprina, G. Pucci
  On the Expansion and Diameter of Bluetooth-Like Topologies, ToCS, 2012