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On the Diameter of Bluetooth-Based Ad Hoc Networks

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Contents

Ad Hoc Networks

- Autonomous distributed systems connected wirelessly
- No centralized control, random placement ⇒ self-organizing, location-oblivious algorithms
- Local data processing ⇒ less traffic on the network
- No addressing ⇒ broadcast multi-hop transmission
- Applications: sensing, control, surveillance, engineering, . . .

Bluetooth Technology

- Suitable for short-range applications (PANs)
- 2.4 GHz ISM band, 1–3 Mb/s rate, low current absorption
- Frequency-hopping with static interference avoidance mechanism
- Piconet: 1 master and up to 7 slaves sharing a common hopping pattern
- The master device switches slave-to-slave transmissions
- Scatternet formation: one slave can belong to more than one piconet

Bluetooth Topology $BT(r(n), c(n))$

Our Reference Model

We model the device discovery phase [IPDPS04] in Bluetooth-based networks with the random graph known as Bluetooth Topology $BT(r(n), c(n))$

Definition (Bluetooth Topology [SPAA04])

$G = (V, E)$ undirected random graph obtained in the following manner:

- spread $|V| = n$ points uniformly at random in $[0, 1]^2$;
- each point selects $c(n)$ neighbours among all the nodes within distance $r(n)$, choosing uniformly at random;
- $\{u, v\} \in E \quad \Leftrightarrow \quad u$ has selected v or vice versa.

Connectivity of $BT(r(n), c(n))$

Theorem (Pietracaprina, Pucci *et al.* [EURO07])

There exist two positive real constants γ_1, γ_2 *such that, if*

$$r(n) \geqslant \gamma_1 \sqrt{\frac{\log n}{n}} \quad \textit{and} \quad c(n) = \gamma_2 \log \frac{1}{r(n)},$$

BT(r(n),c(n)) is connected w.h.p.

- A similar result was demonstrated in [SPAA05] for $r, c = \Theta(1)$
- It exhibits expansion properties [SPAA04] whenever $r, c = \Theta(1)$
- Simulations indicate that $c = 3$ choices are almost always sufficient to obtain connectedness [IPDPS04, EURO07]

The Diameter of Bluetooth Topology

We studied the diameter of $BT(r(n), c(n))$ for those parameter values which ensure connectivity

Definition (Diameter)

$G = (V, E)$ undirected connected graph, the diameter of G is

$$\text{diam}(G) = \max_{u,v \in V} \text{dist}(u, v)$$

where $\text{dist}(u, v)$ is the number of edges in a shortest path between u and v.

- Assuming multi-hop communication with unitary cost per traversed edge:

  diameter $\approx$ maximum delay on the network

- Edge cost $=$ (rx $+$ tx $+$ queueing $+$ processing $+ \ldots$) time

The Main Results: Upper Bound to diam(BT)

Theorem (Upper Bound)

There exist two positive real constants γ_1, γ_2 *such that if*

$$r(n) \geqslant \gamma_1 \sqrt{\frac{\log n}{n}} \quad \textit{and} \quad c(n) = \gamma_2 \log \frac{1}{r(n)}$$

then the following events occur w.h.p.:

1. $BT(r(n), c(n))$ *is connected;*
2. $\mathrm{diam}(BT(r(n), c(n))) = \begin{cases} O\left(\frac{1}{r(n)}\right) & \textit{if } r(n) \leqslant n^{-\frac{1}{8}} \\ O\left(\frac{1}{r(n)} + \log n\right) & \textit{if } r(n) > n^{-\frac{1}{8}} \end{cases}$

The Main Results: Lower Bounds to diam(BT)

Theorem (Geometric Lower Bound)

There exists a positive real constant γ_1 such that if

$$r(n) \geqslant \gamma_1 \sqrt{\frac{\log n}{n}}$$

then w.h.p.

$$\operatorname{diam}(BT(r(n), c(n))) = \Omega\left(\frac{1}{r(n)}\right).$$

Theorem (Improved Lower Bound)

Let $r(n) = \sqrt{2}$ and $c(n) = \Theta(1)$. Then, as $n \to \infty$,

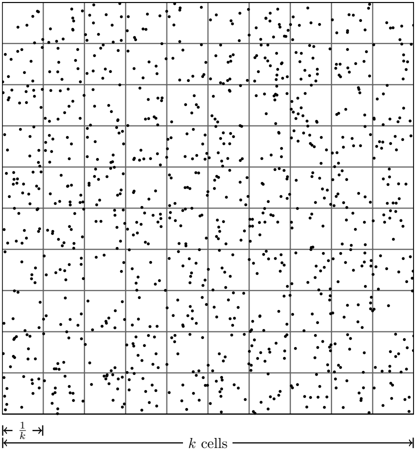
$$\Pr\left[\operatorname{diam}(BT(r(n), c(n))) = \Omega\left(\frac{\log n}{\log\log n}\right)\right] \to 1.$$

Proof Framework

Consider the standard tessellation of $[0,1]^2$ into k^2 square cells, with

$$k = \left\lceil \frac{\sqrt{5}}{r(n)} \right\rceil$$

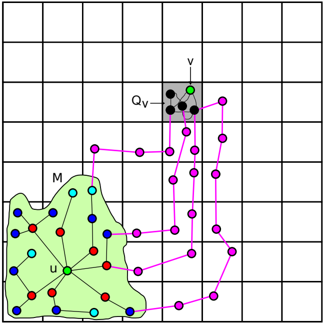
- two nodes residing in adjacent cells are within distance $r(n)$
- w.h.p. each cell contains $\approx \frac{n}{k^2}$ nodes
- w.h.p. each device sees $\approx nr^2$ nodes in its visibility range



UB Case 1 — Short radii: $r(n) \leqslant n^{-\frac{1}{3}}$

- Perform a BFS until $m = \gamma_3 \log n$ nodes are reached (set M in figure).
- From each $w \in M$ start a path with nodes in adjacent cells. Paths have to be edge-disjoint.

  With probability $\geqslant 1 - o\left(\frac{1}{n^3}\right)$ at least one reaches Q_v (internally connected).



Union bound over $O(n^2)$ pairs of nodes $\Rightarrow$

$$\mathrm{diam}(BT) = O(m + 2k + n/k^2) = O\left(\frac{1}{r(n)}\right) \quad \text{w.h.p.}$$

UB Case 2 — Medium radii: $n^{-\frac{1}{3}} < r(n) \leqslant n^{-\frac{1}{8}}$

Similar idea, but now $m = n/k^2 \approx nr^2$ dominates $2k$: we have to bound the diameter of G_Q (still internally connected).

Let $V(Q)$ denote the set of nodes residing in Q, with $|V(Q)| = m$.

- With a BFS argument we can show that, starting from a given $u \in V(Q)$, we can reach $M = \frac{m}{2} + 1$ nodes in $V(Q)$ (the majority) with at most $\log_2 M$ hops with probability $\geqslant 1 - o\left(\frac{1}{m^4}\right)$.
- Applying the union bound over $V(Q)$, we obtain that

$$\forall u, v \in V(Q) \quad \text{dist}(u, v) \leqslant 2\log_2 M + 1 = O(\log n).$$

- With a proper union bound over all $k^2 = O(m^2)$ cells, we have that

$$\Pr\left[\forall \text{cell } Q \quad \text{diam}(G_Q) = O(\log n)\right] \geqslant 1 - o\left(\frac{1}{m}\right) = 1 - o\left(\frac{1}{n^{1/3}}\right).$$

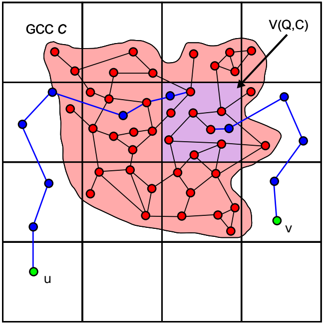
Concluding,

$$\text{diam}(BT) = O\left(\gamma_3 \log n + 2k + O(\log n)\right) = O\left(\frac{1}{r(n)}\right) \quad \text{w.h.p.}$$

UB Case 3 — Long radii: $r(n) > n^{-\frac{1}{8}}$

Now G_Q's are not internally connected but a GCC C with $\geqslant \frac{n}{8k^2}$ nodes exists w.h.p.

- Perform a BFS until $m = n^{1/3}$ nodes are reached. We can upperbound the real process setting $c(n) = 2$ obtaining w.h.p. a complete binary tree of height $\log_2 m = \Theta(\log n)$.
- As before, with probability $\geqslant 1 - o\left(\frac{1}{n^3}\right)$ there exist at least two paths starting from u and v leading to $V(Q, C)$.
- We proved that $\text{diam}(GCC) = O(\log n)$ with a branching-process argument.



Union bound over $O(n^2)$ pairs of nodes $\Rightarrow$

$$\text{diam}(BT) = O(2(\gamma_3 \log n + 2k) + \log n) = O\left(\frac{1}{r(n)} + \log n\right) \quad \text{w.h.p.}$$

Conclusions and future work

Our results

We demonstrated tight asymptotic results about the diameter of $BT(r(n), c(n))$ for almost all the radius values, especially for the interesting ones, i.e. $r(n) \approx r_{MIN}$

Possible improvements

- $c = \Theta(1)$ suffices for connectivity
- Different node distributions (holes, different area shapes)
- Mobility issues
- Neighbour selection as a stochastic process in time, other selection policies
- Routing algorithms
- ...

Q&A

Thanks for your attention!

Any question?

References

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Geometric Lower Bound

Theorem (Geometric Lower Bound)

There exists a positive real constant γ_1 *such that if*

$$r(n) \geqslant \gamma_1 \sqrt{\frac{\log n}{n}}$$

then w.h.p.

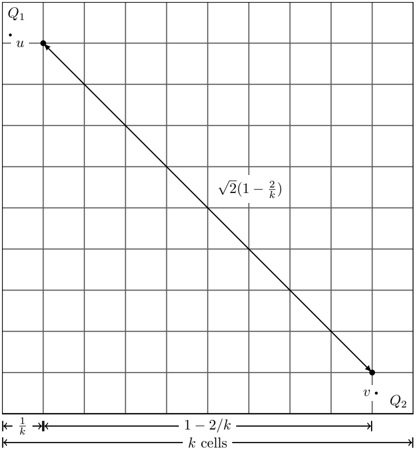
$$\mathrm{diam}(BT(r(n), c(n))) = \Omega\left(\frac{1}{r(n)}\right).$$

Proof.

We need at least

$$\left\lceil \frac{\sqrt{2}\left(1-\frac{2}{k}\right)}{r} \right\rceil = \Omega\left(\frac{1}{r(n)}\right)$$

hops to move from u to v. □



Improved Lower Bound

Theorem (Improved Lower Bound)

Let $r(n) = \sqrt{2}$ *and* $c(n) = \Theta(1)$. *Then, as* $n \to \infty$,

$$\Pr\left[\mathrm{diam}(BT(r(n), c(n))) = \Omega\left(\frac{\log n}{\log\log n}\right)\right] \to 1.$$

Proof.

Every device can select every other node. Let X_u be the degree of u: $\mathrm{E}[X_u] = 2c$.

Using a Chernoff bound:

$$\Pr\left[\forall u \quad X_u \leqslant 3\log n + c = d\right] \geqslant 1 - o\left(\frac{1}{n^2}\right).$$

The diameter of BT cannot be less than the diameter of a tree with ariety $d - 1$.

Therefore, $\mathrm{diam}(BT) = \Omega(\log_{d-1} n) = \Omega\left(\frac{\log n}{\log\log n}\right)$ w.h.p. □